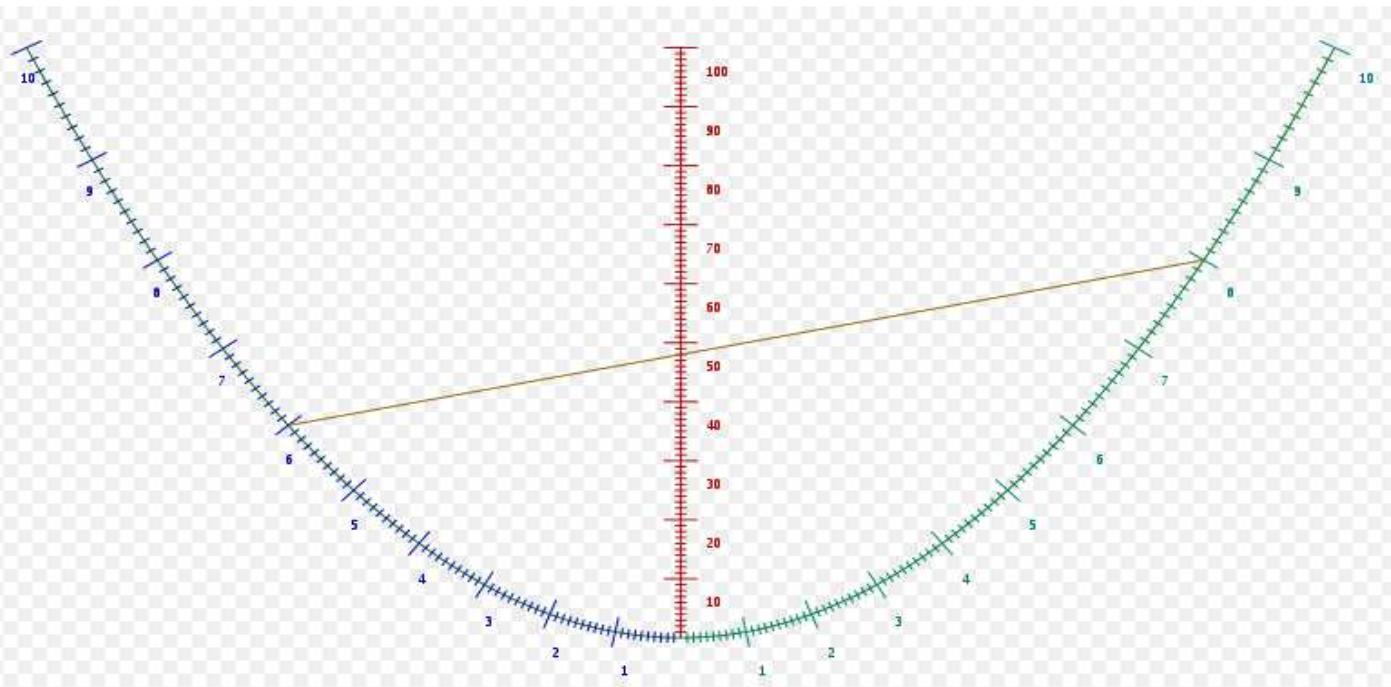


The multiplication parabola

A nomogram, also called an abacus, is a graphical calculating device designed to allow the approximate graphical computation of a function.

The following nomogram is based on the graph of a parabola.

Choose point 6 on one half of the parabola and point 8 on the other half of the parabola. Then draw the line and note the intersection point with the vertical axis : that's 48... which is 6×8 !



Adapted from wikipedia

Tasks

1. Choose two integers and verify that their product can be found on the above graph.
2. Consider the parabola $y = x^2$. State the properties of this elementary function and the properties of its graph.
3. Let a and b be two distinct real numbers.
 - a) Compute the slope of the line that crosses the parabola $y = x^2$ where $x = a$ and where $x = b$ on the parabola.
 - b) Deduce the equation of the line.
 - c) Compute its y -intercept.
 - d) Conclude.
4. Explain the construction of the nomogram from the graph of the parabola $y = x^2$.
5. Let's consider two integers a and b greater than or equal to 2 on both sides of the parabola, join them and look closely at the intersection point with the vertical axis.
 - a) Explain why 3 and 5 will not be reached.
 - b) Give some examples of other integers that will not be reached.
 - c) Give the major arithmetical property that all these integers share.

The multiplication parabola
Comments and answers

1. Choose any pair of integers on both halves of the parabola : the line crosses the y -axis at the value of their product !
2. The parabola $y = x^2$ is the graph of the squaring function, which is an even function, with domain \mathbb{R} (set of all real numbers) and range the interval $[0; + \infty)$.
It is symmetrical about the y -axis, in a U-shape (or upwards), with the vertex at point $(0,0)$.
3. Let's consider a and b be two distinct real numbers.
 - a) Let's call A the point $(a; a^2)$ and B the point $(b; b^2)$.

The gradient of line (AB) is :
$$\frac{y_B - y_A}{x_B - x_A} = \frac{b^2 - a^2}{b - a} = \frac{(b - a)(b + a)}{b - a} = b + a.$$
 - b) Then the equation of line (AB) is : $y - a^2 = (a + b)(x - a)$ or $y - b^2 = (a + b)(x - b)$, or $y = (a + b)x - ab$.
 - c) The y -intercept is the value of y when $x = 0$ which is here $-ab$.
 - d) We have proved that the y -intercept of the line joining two points on the graph of the parabola $y = x^2$ is the opposite of the product of their abscissas.
4. To build the nomogram from the graph of the parabola $y = x^2$, we have to mark absolute values of abscissas on each half of the parabola, both sides of the y -axis.
5. a) If a and b are both greater than or equal to 2, then, their product is an integer greater than or equal to 4. Then 3 can't be reached on the vertical axis. The integer 5 can't be reached as well because 5 is not the product of two integers.
b) That's also the case of 7, 11, 13, 17...etc which are all integers that are not the product of two integers.
c) Such integers are called prime numbers that are divisible only by 1 and themselves.